

Excited decuplet baryons from QCD sum rules

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A calculation of the mass spectrum in the baryon decuplet sector is presented using the method of QCD sum rules. Sum rules are derived for states of spin-parity $3/2\pm$ and $1/2+$ using both the conventional method and a parity-projection method. The predictive ability of the sum rules is explored by a Monte-Carlo based analysis procedure in which the three phenomenological parameters (mass, coupling, threshold) are treated as free parameters and fitted simultaneously. Taken together, the results give an improved determination of the mass spectrum in this sector from the perspective of non-perturbative QCD.

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I. INTRODUCTION

A goal of hadronic physics is to understand the baryon spectrum from QCD, the underlying theory of the strong interaction. Experimentally, the push is fueled by the physics program at JLab, and other accelerator facilities. To this end, the time-honored QCD sum rule method [1] adds a useful theoretical tool. It is a non-perturbative approach to QCD that reveals a direct connection between hadronic observables and the QCD vacuum structure via a few universal parameters called vacuum condensates (vacuum expectation values of QCD local operators), the most important of which are the quark condensate, the mixed condensate and the gluon condensate. The method is analytical, physically transparent, and has minimal model dependence with well-understood limitations inherent in the operator product expansion (OPE). It provides a complementary view of the same non-perturbative physics to the numerical approach of lattice QCD. The method was applied to the decuplet sector not long after it was introduced [2, 3, 4, 5, 6]. However, only limited attention has been paid to this sector since then. In Ref. [7], a systematic study was made that includes some Δ states. Later, progress came in the analysis of QCD sum rules by Monte-Carlo sampling of errors [8]. It was applied to the study of spin-3/2 branch of baryon decuplet [9], magnetic moments [10, 11] and other spin-3/2 excited baryons [12].

From a theoretical standpoint, the main issue is how to isolate sum rules that couple to a particular spin-parity, since the interpolating fields used to construct the spin-3/2 states contain both spin-3/2 and spin-1/2 components, and couple to both parities. Separation of the spin-3/2 and spin-1/2 components can be achieved by examining the Dirac structures upon which the sum rules are based, as done in Ref. [7]. However, the parities are still mixed. In the pole-plus-continuum ansatz of the method, isolation of a particular parity relies on strong cancellations in the excited part of the spectrum. Without such cancellations, the sum rules usually suffer contaminations from higher states, a problem that renders them unstable. A solution to separate the parities exactly

was proposed in Ref. [13] which showed improved convergence in the octet baryon sector. In this work, we apply the method to the decuplet sector, together with the conventional method. The goal is to identify sum rules that couple strongly to the four branches of the decuplet with spin-parity $3/2+$, $3/2-$, $1/2+$, and $1/2-$. In addition to the Delta states, we also examine states that contain the strange quark, which have not been studied in detail. In our calculation, we consistently include operators up to dimension eight, first order strange quark mass corrections, flavor symmetry breaking of the strange quark condensates, anomalous dimension corrections, and possible factorization violation of the four-quark condensate. Furthermore, we try to assess quantitatively the errors in the phenomenological parameters, using the Monte-Carlo based analysis procedure. This procedure incorporates all uncertainties in the QCD input parameters simultaneously, and translates them into uncertainties in the phenomenological parameters, with careful regard to OPE convergence and ground state dominance.

II. METHOD

In the conventional method, the starting point is the time-ordered two-point correlation function in the QCD vacuum

$$\Pi(p) = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ \eta(x) \bar{\eta}(0) \} | 0 \rangle. \quad (1)$$

where η is the interpolating field with the quantum numbers of the baryon under consideration. Assuming SU(2) isospin symmetry in the u and d quarks, we consider the most general current of spin 3/2 and isospin 3/2 for the Δ ,

$$\eta_\mu^\Delta(x) = \epsilon^{abc} [u^{aT}(x) C \sigma_\mu u^b(x)] u^c(x) \quad (2)$$

Here C is the charge conjugation operator and the superscript T means transpose. The antisymmetric ϵ^{abc} and sum over color ensures a color-singlet state. For the other members of the decuplet, we consider, omitting the

explicit x -dependence

$$\eta_{\mu}^{\Sigma*} = \sqrt{1/3}\epsilon^{abc}\{2[u^{aT}C\sigma_{\mu}s^b]u^c + [u^{aT}C\sigma_{\mu}u^b]s^c\}, \quad (3)$$

$$\eta_{\mu}^{\Xi*} = \sqrt{1/3}\epsilon^{abc}\{2[s^{aT}C\sigma_{\mu}u^b]s^c + [s^{aT}C\sigma_{\mu}s^b]u^c\}, \quad (4)$$

$$\eta_{\mu}^{\Omega^-} = \epsilon^{abc}[s^{aT}C\sigma_{\mu}s^b]s^c. \quad (5)$$

A baryon interpolating field couples to both the ground state and the excited states of a baryon, and to both parities. That leads to sum rules that contain mixed parity states. The parities can be separated by considering the ‘forward-propagating’ version of the correlation function [13],

$$\Pi(p) = i \int d^4x e^{ip \cdot x} \theta(x_0) \langle 0 | T \{ \eta(x) \bar{\eta}(0) \} | 0 \rangle. \quad (6)$$

where the only difference between the above equation and the conventional two-point correlation function in Eq. (1) is the step function $\theta(x_0)$. Under this condition, the phenomenological representation of the the imaginary part in the rest frame ($\vec{p}=0$) can be written as

$$\begin{aligned} \text{Im } \Pi(p_0) &= \sum_n \left[(\lambda_n^+)^2 \frac{\gamma_0 + 1}{2} \delta(p_0 - m_n^+) \right. \\ &\quad \left. + (\lambda_n^-)^2 \frac{\gamma_0 - 1}{2} \delta(p_0 - m_n^-) \right] \\ &\equiv \gamma_0 A(p_0) + B(p_0) \end{aligned} \quad (7)$$

where λ^{\pm} and m^{\pm} are the couplings and masses of the states involved, separated by parity. The functions in the last step are defined by

$$A(p_0) = \frac{1}{2} \sum_n [(\lambda_n^+)^2 \delta(p_0 - m_n^+) + (\lambda_n^-)^2 \delta(p_0 - m_n^-)], \quad (8)$$

and

$$B(p_0) = \frac{1}{2} \sum_n [(\lambda_n^+)^2 \delta(p_0 - m_n^+) - (\lambda_n^-)^2 \delta(p_0 - m_n^-)]. \quad (9)$$

By considering the combinations $A(p_0) + B(p_0)$ and $A(p_0) - B(p_0)$, positive-parity and negative-parity states are separated exactly, respectively. This method is reminiscent of the one used in lattice QCD under Dirichlet boundary conditions in the time direction [14].

The construction of sum rules in Borel space proceeds by taking the integral with a weighting factor and truncating all excited states starting at a certain threshold. For positive-parity states,

$$\int_0^{w^+} [A(p_0) + B(p_0)] e^{-p_0^2/M^2} dp_0 = \lambda_+^2 e^{-m_+^2/M^2} \quad (10)$$

and for positive-parity states,

$$\int_0^{w^-} [A(p_0) - B(p_0)] e^{-p_0^2/M^2} dp_0 = \lambda_-^2 e^{-m_-^2/M^2}. \quad (11)$$

To separate out the spin components, we need the full Dirac structure of the correlation function for spin-3/2 interpolating fields [7, 9],

$$\begin{aligned} \Pi_{\mu\nu}(p) &= \lambda_{3/2}^2 \left\{ -g_{\mu\nu} \hat{p} + \frac{1}{3} \gamma_{\mu} \gamma_{\nu} \hat{p} \right. \\ &\quad \left. - \frac{1}{3} (\gamma_{\mu} p_{\nu} - \gamma_{\nu} p_{\mu}) + \frac{2}{3} \frac{p_{\mu} p_{\nu}}{M_{3/2}^2} \right. \\ &\quad \left. \pm M_{3/2} \left[g_{\mu\nu} - \frac{1}{3} \gamma_{\mu} \gamma_{\nu} \right. \right. \\ &\quad \left. \left. + \frac{1}{3M_{3/2}^2} (\gamma_{\mu} p_{\nu} - \gamma_{\nu} p_{\mu}) \hat{p} - \frac{2}{3} \frac{p_{\mu} p_{\nu}}{M_{3/2}^2} \right] \right\} \\ &\quad + \alpha_{1/2}^2 \left\{ \frac{16}{M_{1/2}^2} p_{\mu} p_{\nu} \hat{p} - \gamma_{\mu} \gamma_{\nu} \hat{p} \right. \\ &\quad \left. - 6 (\gamma_{\mu} p_{\nu} + \gamma_{\nu} p_{\mu}) + 4 (\gamma_{\mu} p_{\nu} - \gamma_{\nu} p_{\mu}) \right. \\ &\quad \left. \pm M_{1/2} \left[-\gamma_{\mu} \gamma_{\nu} - \frac{8 p_{\mu} p_{\nu}}{M_{1/2}^2} \right. \right. \\ &\quad \left. \left. + \frac{4}{M_{1/2}^2} (\gamma_{\mu} p_{\nu} - \gamma_{\nu} p_{\mu}) \hat{p} \right] \right\} + \dots, \end{aligned} \quad (12)$$

where the ellipses denote excited state contributions, and \hat{p} denotes $p_{\mu} \gamma^{\mu}$. The upper/lower sign corresponds to the explicit positive/negative parity when a state contributes. The entire structure is divided into two parts: one for spin-3/2 with coupling $\lambda_{3/2}^2$, and the other spin-1/2 with coupling $\alpha_{1/2}^2$. The structures are chosen to be effectively orthogonal to each other. Only six of the structures are independent. The two sum rules from $g_{\mu\nu} \hat{p}$ and $g_{\mu\nu}$ couple only to spin-3/2 states because they do not appear in the spin-1/2 part. The two sum rules from $\gamma_{\mu} p_{\nu} + \gamma_{\nu} p_{\mu}$ and $\gamma_{\mu} \gamma_{\nu} + \frac{1}{3} g_{\mu\nu}$ couple only to spin-1/2 states because they do not appear in the spin-3/2 part. By the same token, the sum rule at $\gamma_{\mu} p_{\nu} + \gamma_{\nu} p_{\mu}$ that couples to spin-1/2 could also be obtained from the combination $\gamma_{\mu} \gamma_{\nu} \hat{p} + \frac{1}{3} g_{\mu\nu} \hat{p}$. The two sum rules from $p_{\mu} p_{\nu} \hat{p}$ and $p_{\mu} p_{\nu}$ couple to both spin-3/2 and spin-1/2 states because they appear in both parts. They will not be considered further. Note that the parities are still mixed in each of these sum rules. It is not known *a priori* which parity is the dominant state in a given sum rule. That information comes from the specific OPE structure on the left-hand-side.

III. PARITY-PROJECTED QCD SUM RULES FOR SPIN-3/2 DECUPLET STATES

The combination of the two tensor structures that couple purely to spin-3/2 states, $g_{\mu\nu} \hat{p}$ and $g_{\mu\nu}$, can be cast

into the form $g_{\mu\nu}(\gamma_0 A + B)$ in the rest frame. According to Eq. (7), it suggests that parity projection can be done exactly in this case. The functions $A(p_0)$ and $B(p_0)$ are readily identified from the usual calculation of the OPE which involves contracting out the quark pairs in the correlation function and substituting the fully-interacting quark propagator from OPE. They are functions of the QCD vacuum condensates and other QCD parameters.

Upon Borel integration in Eq. (10) and Eq. (11), the parity-projected sum rules can be written in the general form

$$\mathcal{A}(w_+, M) + \mathcal{B}(w_+, M) = \tilde{\lambda}_+^2 e^{-m_+^2/M^2}, \quad (13)$$

$$\mathcal{A}(w_-, M) - \mathcal{B}(w_-, M) = \tilde{\lambda}_-^2 e^{-m_-^2/M^2}, \quad (14)$$

where $\tilde{\lambda}_\pm = (2\pi)^2 \lambda_\pm$ are the rescaled couplings. The rescaling is done so that no factors of π appear explicitly in the sum rules. The rescaled quantities are also ‘natural’ in their numerical values.

For our special case of spin-3/2 states, the \mathcal{A} function $\mathcal{A}_{3/2}$ is given by

$$\begin{aligned} \mathcal{A}_{3/2} = & c_1 [1] (2(1 - e^{-w^2/M^2})M^6 - 2w^2 e^{-w^2/M^2} M^4 \\ & - w^4 e^{-w^2/M^2} M^2) L^{4/27} \\ & + (c_2 [b] + c_3 [m_s a]) (1 - e^{-w^2/M^2}) M^2 L^{4/27} \\ & + c_4 [m_s m_0^2 a] L^{-10/27} + c_5 [\kappa_v a^2] L^{28/27}. \end{aligned} \quad (15)$$

In this expression, the vacuum condensates are explicitly isolated in square brackets whose definitions and values will be given below. The Wilson coefficients (which are pure numbers) are given for various members of the decuplet by

$\mathcal{A}_{3/2}$	c_1	c_2	c_3	c_4	c_5
Δ	$\frac{1}{20}$	$\frac{-5}{144}$	0	0	$\frac{-2}{3}$
Σ^*	$\frac{1}{20}$	$\frac{-5}{144}$	$\frac{(4-f_s)}{6}$	$\frac{-(14-5f_s)}{36}$	$\frac{(2+4f_s)}{9}$
Ξ^*	$\frac{1}{20}$	$\frac{-5}{144}$	$\frac{(2+f_s)}{3}$	$\frac{-(7+2f_s)}{18}$	$\frac{2f_s(2+f_s)}{9}$
Ω^-	$\frac{1}{20}$	$\frac{-5}{144}$	$\frac{f_s}{2}$	$\frac{-3f_s}{4}$	$\frac{-2f_s^2}{3}$

(16)

Similarly, the $\mathcal{B}_{3/2}$ function is given by

$$\begin{aligned} \mathcal{B}_{3/2} = & c_1 [m_s] (I(w) - w e^{-w^2/M^2}) L^{-8/27} M^4 \\ & + c_2 [m_s] w^3 e^{-w^2/M^2} L^{-8/27} M^2 \\ & + c_3 [a] (I(w) - w e^{-w^2/M^2}) L^{16/27} M^2 \\ & + c_4 [m_s b] I(w) L^{-8/27} \\ & + c_5 [m_0^2 a] I(w) L^{2/27}, \end{aligned} \quad (17)$$

where the coefficients are given by

$\mathcal{B}_{3/2}$	c_1	c_2	c_3	c_4	c_5
Δ	0	0	$\frac{2}{3}$	0	$\frac{-2}{3}$
Σ^*	$3/16$	$-1/8$	$\frac{2(2+f_s)}{9}$	$-1/24$	$\frac{-2(2+f_s)}{9}$
Ξ^*	$3/8$	$-1/4$	$\frac{2(1+2f_s)}{9}$	$-1/12$	$\frac{-2(1+2f_s)}{9}$
Ω^-	$9/16$	$-3/8$	$\frac{2f_s}{3}$	$-1/8$	$\frac{-2f_s}{3}$

(18)

The integral $I(w) = \int_0^w e^{-x^2/M^2} dx$ is evaluated numerically in the analysis. Note that the sum rule for Ω^- reduces to that for the Δ if $m_s = 0$ and $f_s = 1$, which serves as a check of the calculation.

Now we explain the meaning of the parameters in the sum rules. The rescaled quark condensate is taken as the positive quantity $a = -(2\pi)^2 \langle \bar{u}u \rangle = 0.52 \pm 0.05 \text{ GeV}^3$, corresponding to a central value of $\langle \bar{u}u \rangle = -(236)^3 \text{ MeV}^3$. For the gluon condensate, we use rescaled $b = \langle g_c^2 G^2 \rangle = 1.2 \pm 0.6 \text{ GeV}^4$. The mixed condensate parameter is placed at $m_0^2 = \langle \bar{q}g\sigma \cdot Gq \rangle / \langle \bar{q}q \rangle = 0.72 \pm 0.08 \text{ GeV}^2$. For the four-quark condensate $\langle \bar{u}u\bar{u}u \rangle = \kappa_v \langle \bar{u}u \rangle^2$, we use $\kappa_v = 2 \pm 1$ to include possible violation of the factorization approximation. We retain only terms linear in the strange quark mass and set $m_u = m_d = 0$. The strange quark mass is taken as $m_s = 0.15 \pm 0.02 \text{ GeV}$. We use the ratio $f_s = \langle \bar{s}s \rangle / \langle \bar{u}u \rangle = \langle \bar{s}g_c\sigma \cdot Gs \rangle / \langle \bar{u}g_c\sigma \cdot Gu \rangle = 0.83 \pm 0.05$ to account for the flavor symmetry breaking of the strange quark condensates. The anomalous dimension corrections of the various operators are taken into account via the factors $L^\gamma = [\alpha_s(\mu^2)/\alpha_s(M^2)]^\gamma = [\ln(M^2/\Lambda_{QCD}^2)/\ln(\mu^2/\Lambda_{QCD}^2)]^\gamma$, where γ is the appropriate anomalous dimension, $\mu = 500 \text{ MeV}$ is the renormalization scale, and $\Lambda_{QCD} = 0.15 \pm 0.04 \text{ GeV}$ is the QCD scale parameter. We find variations of Λ_{QCD} have little effects on the results. The uncertainties are assigned fairly conservatively, ranging from 10% to 100%. They will be mapped into those for the fit parameters in the analysis, giving a realistic estimate of the errors on the parameters.

It is worth pointing out that the A function is chiral-even, meaning that it only involves condensates of even energy dimensions: for example, b of 4, $m_s a$ of 4, $\kappa_v a^2$ of 6 and so on. On the other hand, the B function is chiral-odd, with the leading contribution from the quark condensate a , the order parameter of spontaneous chiral symmetry breaking of QCD. The difference of the two parity-projected sum rules in Eq. (13) and Eq. (14) is the sign in front of the chiral-odd term B . So from the point of view of QCD sum rules, the origin of splittings between positive-parity and negative-parity states lies in the chiral-odd vacuum condensates. If the quark condensate and the strange quark mass vanish, then chiral symmetry is restored in the vacuum, and there would be exact parity doubling in the baryon spectrum. This is a valuable insight from the parity-projected QCD sum rules.

The analysis of a QCD sum rule boils down to the following mathematical problem. Given an equation of the general structure

$$\text{LHS}(M, OPE) = \text{RHS}(M, w, m, \lambda^2), \quad (19)$$

and a set of input QCD parameters denoted by OPE , find the best output parameters (the baryon mass of interest m , the coupling strength λ^2 of the interpolating field, and the continuum threshold w) by matching the two sides over a region in Borel mass M . The LHS has errors arising from our imprecise knowledge of the QCD

parameters, as discussed above. From statistical point of view, a χ^2 minimization of the type

$$\chi^2 = \sum_i \frac{|\text{LHS}_i - \text{RHS}_i|^2}{\sigma_i^2}, \quad (20)$$

offers the least-biased way of finding the unknown parameters. In the Monte-Carlo based procedure used here, first the Borel window in M is divided equally into a grid (we use 51 points). At each point, the uncertainty distribution in the OPE is constructed by randomly-selected, Gaussianly-distributed sets generated from the central values and the uncertainties in the QCD input parameters. Then the χ^2 minimization is applied to the sum rule by fitting the phenomenological parameters. This is done for each QCD parameter set and at each point in M , resulting in distributions for phenomenological fit parameters, from which their errors are derived. Usually, 100 such configurations are sufficient for getting stable results. We generally select 500 sets which help resolve more subtle correlations among the QCD parameters and the phenomenological fit parameters. So the same sum rule is fitted not just once, but thousands of times in our analysis. The advantage afforded by the Monte-Carlo is that the entire phase-space of the input QCD parameters is explored. The errors obtained this way represent the most realistic and conservative estimates of the predictive power of a QCD sum rule. This is in contrast to traditional approaches where only a small part of the phase-space is explored at a time. We select the Borel window by trial and error. Our criteria are that the OPE is reasonably convergent by looking at the terms of various dimension, and that the fit results should not be sensitive to small changes in the Borel window. Under these constraints, we seek to make the window as wide as possible. Since the selection of Borel window M and the three phenomenological parameters are inter-dependent in the sum rule, the entire fitting process is iterated until the best solution is found. In general, we seek solutions that are from a three-parameter search in which all three parameters are treated as free simultaneously. We regard such solutions as the best predictions of the QCD sum rule approach. In the absence of such solutions, a two-parameter search is performed in which one of the three parameters is fixed. Usually, it is the continuum threshold which is fixed to values that are larger than the mass in question, or to values suggested by the observed spectrum. Such a two-parameter approach is the one usually adopted in most analyses in the past.

Fig. 1 shows the matching in the sum rule Eq. (13) for $3/2+$ states. The two sides match very well over a relatively wide region (1 GeV). The individual contributions of term \mathcal{A} and term \mathcal{B} are also shown. The two terms are comparable in size, the sum of which gives rise to the LHS. The error band on the LHS is generated by Monte-Carlo reflecting all the uncertainties assigned to the QCD parameters. The entire uncertainty phase space is then mapped to the output parameters in the fitting process. A three-parameter search was tried first. Unfortunately,

TABLE I: Results for the $3/2+$ states from the parity-projected sum rule Eq.(13) in a two-parameter search. The errors are derived from 500 Monte-Carlo sets based on the uncertainties assigned to the QCD input parameters. Two sets of solutions are provided for each case: the first one using the default errors in the OPE input parameters, while the second using uniform 10% errors. The experimental values are taken from the PDG [15].

	Region	w	$\tilde{\lambda}_{3/2+}^2$	Mass	Exp.
	(GeV)	(GeV)	(GeV ⁶)	(GeV)	(GeV)
$\Delta(\frac{3}{2}+)$	1.4 to 2.4	1.60	2.46 ± 0.42	1.21 ± 0.06	1.23
			2.32 ± 0.21	1.21 ± 0.02	
$\Sigma^*(\frac{3}{2}+)$	1.4 to 2.4	1.84	3.52 ± 0.43	1.38 ± 0.06	1.385
			3.40 ± 0.29	1.37 ± 0.02	
$\Xi^*(\frac{3}{2}+)$	1.4 to 2.4	1.95	4.19 ± 0.46	1.47 ± 0.05	1.53
			4.10 ± 0.37	1.47 ± 0.02	
$\Omega^-(\frac{3}{2}+)$	1.4 to 2.4	2.25	6.94 ± 0.63	1.68 ± 0.05	1.67
			6.89 ± 0.56	1.68 ± 0.02	

a solution could not be found. What happens is that the search algorithm keeps returning solutions with threshold smaller than the mass, a clearly unphysical situation. This is an indication that the OPE does not have enough information to resolve all three parameters simultaneously. So we switched to a two-parameter search by fixing the continuum threshold to values suggested by the Particle Data Group [15]. The extracted parameters are given in Table I. We offer two sets of results on the fit parameters: one using the default errors assigned to the OPE parameters ranging from 10% to 100%, the other with 10% uniform errors. It sort of gives the worst-case, best-case scenarios, as far as errors on the input parameters are concerned. In the worst-case solution, the errors on the masses are on the order of 5%, and 15% on the couplings. In the best-case solution, the errors are reduced to about 2% on the masses, and less than 10% on the couplings. Note that the central value for the couplings are shifted by a small amount when the errors are reduced, while it is stable for the masses. The computed masses compare favorably with experiment. What is interesting is the fact that the mass pattern emerges under the same Borel window across the particles, and that the continuum thresholds are consistent with the excited states suggested by PDG. In the case of $\Xi^*(3/+)$, the computed mass of 1.47 GeV is slightly below the experimental value of 1.53 GeV. In the PDG, there are two 3-star excited states at 1690 MeV and 1950 MeV with unknown spin-parity. Our sum rule favors a threshold of 1950 MeV to 1690 MeV which leads to a mass that is even smaller than 1.47 GeV. It hints that the spin-parity of 1690 MeV is likely not $3/2+$.

Fig. 2 shows the matching in the sum rule Eq. (14) for $3/2-$ states. Here it is the difference between term \mathcal{A} and term \mathcal{B} that gives rise to the ground-state pole. The quality of the matching is still fairly good, but we found that the stability of the sum rules is not as good as the

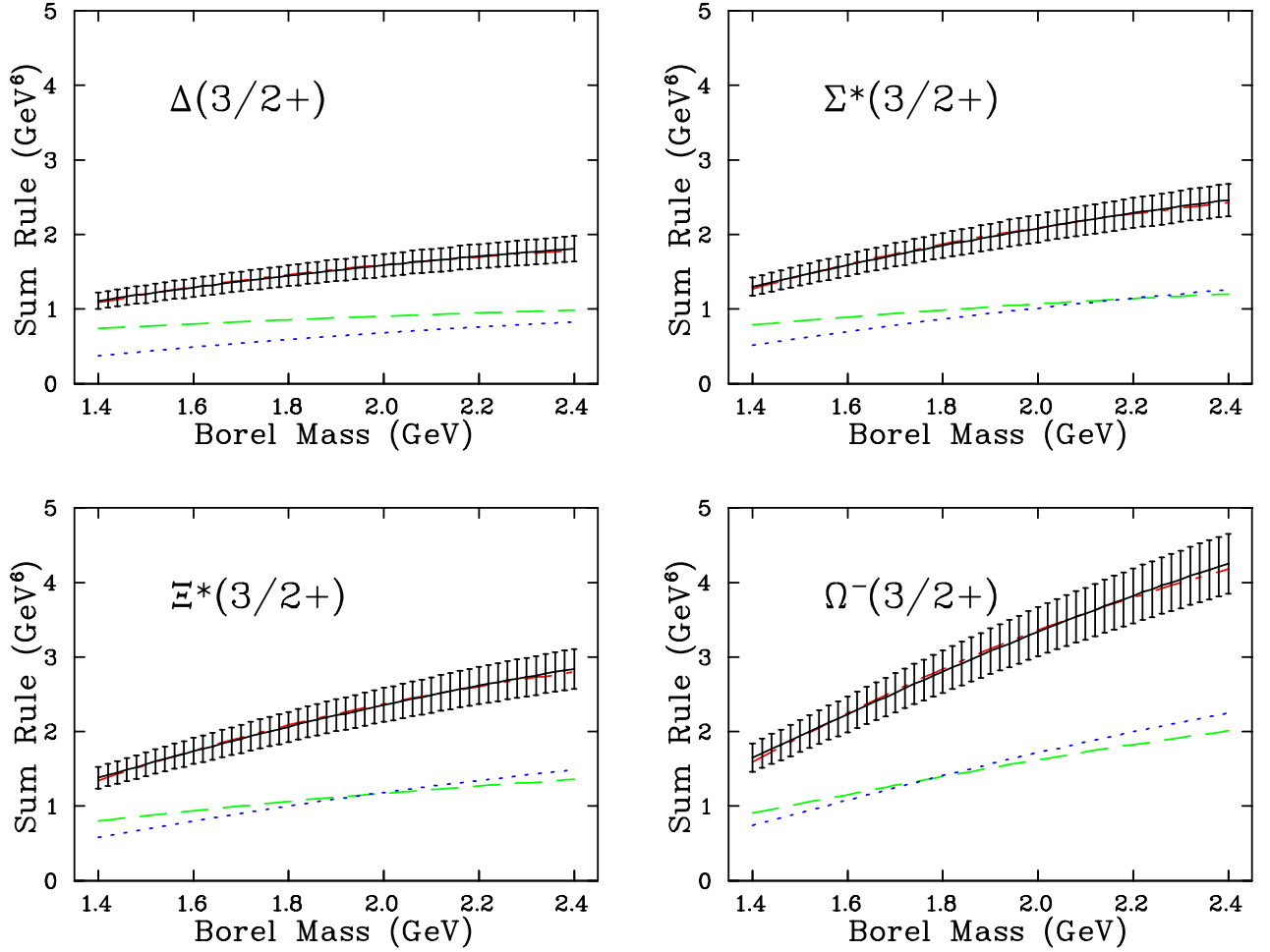


FIG. 1: Matching of the sum rules in Eq. (13) for positive-parity spin-3/2 states as a function of the Borel mass. The solid line along with the error band generated via Monte-Carlo is the LHS ($\mathcal{A} + \mathcal{B}$). The dash-dot line, which is barely distinguishable from the solid line, is the RHS (ground-state pole). Also plotted are the individual contributions of term \mathcal{A} (dotted line) and term \mathcal{B} (dashed line).

corresponding ones for $3/2+$ states in Eq. (13). A three-parameter search does not work, so a two-parameter search is performed. To get a better understanding of the sum rules, we adopt the following fitting strategy. Instead of fixing the continuum threshold to a value that leads to a known state, we perform two-parameter searches by fixing the mass to values suggested by the PDG, and search for the continuum threshold and coupling. In this way, the searches will provide information on whether a specific sum rule can accommodate a known state with reasonable continuum thresholds and couplings. Such a study is useful since to our knowledge there is no information about these $3/2-$ states from the standpoint of QCD sum rules. The result of the study is given in Table II. We fix the Borel window to be the same wide window (1.4 to 2.4 GeV) as for the $3/2+$ states for all cases. This choice is quite reasonable judging by the matching plots, which are all of comparable quality as Fig. 2. Again, the same worst-case, best-case solutions are provided for each case in order to give some

idea about the stability of the fits.

In the case of $\Delta(3/2-)$, the PDG lists a 4-star state at 1.70 GeV. Using this value as input, a continuum threshold of 2.74 GeV and a coupling of 1.56 are obtained, with relative errors about 20% and 55%, respectively. They decrease to about 5% and 40% in the best-case (10% uniform errors on the input parameters). Note that the central values vary as errors are reduced: the threshold shifts up to 2.9 GeV and the coupling shifts down to 1.43. It is a sign of highly non-linear mapping of the errors from input to output. These errors are much bigger than those for $3/2+$ states, especially on the couplings. The next $\Delta(3/2-)$ state in the PDG is a one-star state at 1.94 GeV, compared to the continuum threshold of 2.7 to 2.9 GeV from this sum rule.

In the case of $\Sigma^*(3/2-)$, the PDG lists a 2-star state at 1.58 GeV with unknown spin-parity, a 4-star state at 1.67 GeV, and a three-star state at 1.94 GeV. The corresponding continuum thresholds required to predict these states are 2.87, 2.90, 3.00 GeV.

TABLE II: Results for the $3/2^-$ states from the parity-projected sum rule Eq.(14) in a two-parameter search. The errors are derived from 500 Monte-Carlo sets based on the uncertainties assigned to the QCD input parameters. Two sets of solutions are provided for each case: the first one using the default errors in the OPE input parameters, while the second using uniform 10% errors. The experimental values are taken from the PDG [15].

	Region	w	$\tilde{\lambda}_{3/2^-}^2$	Mass	Exp.
	(GeV)	(GeV)	(GeV ⁶)	(GeV)	(GeV)
$\Delta(\frac{3}{2}^-)$	1.4 to 2.4	2.74 \pm 0.47	1.56 \pm 0.84	1.70	1.70
		2.90 \pm 0.10	1.43 \pm 0.56		
$\Sigma^*(\frac{3}{2}^-)$	1.4 to 2.4	2.68 \pm 0.51	1.15 \pm 0.64	1.58	1.58(?)
		2.87 \pm 0.13	1.02 \pm 0.39		
		2.74 \pm 0.46	1.31 \pm 0.72	1.67	1.67
		2.90 \pm 0.11	1.19 \pm 0.46		
		2.94 \pm 0.30	2.14 \pm 1.20	1.94	1.94
		3.00 \pm 0.06	1.94 \pm 0.76		
$\Xi^*(\frac{3}{2}^-)$	1.4 to 2.4	2.72 \pm 0.43	1.13 \pm 0.65	1.69	1.69(?)
		2.91 \pm 0.12	1.02 \pm 0.42		
		2.86 \pm 0.36	1.40 \pm 0.81	1.82	1.82
		2.95 \pm 0.10	1.28 \pm 0.53		
		2.94 \pm 0.30	1.80 \pm 1.05	1.95	1.95(?)
		3.00 \pm 0.10	1.63 \pm 0.68		
$\Omega^-(\frac{3}{2}^-)$	1.4 to 2.4	-	-	2.25	2.25(?)
		3.11 \pm 0.13	2.31 \pm 1.11		
		3.18 \pm 0.24	3.47 \pm 2.27	2.38	2.38(?)
		3.18 \pm 0.13	3.13 \pm 1.50		
		3.24 \pm 0.24	4.32 \pm 2.77	2.47	2.47(?)
		3.24 \pm 0.14	3.91 \pm 1.82		

In the case of $\Xi^*(3/2^-)$, out of the three 3-star states listed by the PDG, 1.69, 1.82, 1.95 GeV, only the 1.82 GeV state is assigned the $3/2^-$ spin-parity. The corresponding continuum thresholds required to predict these states are 2.91, 2.95, 3.00 GeV. Interestingly, these values are close to the ones for the $\Sigma^*(3/2^-)$ states.

In the case of $\Omega^-(3/2^-)$, there are three candidates from the PDG, a three-star state at 2.25 GeV, a two-star state at 2.38 GeV, and a two-star state at 2.47 GeV, all with unknown spin-parity. Using the sum rules, they could be assigned $3/2^-$ if the corresponding continuum thresholds are 3.11, 3.18, 3.24 GeV. For the 2.25 GeV state, we could find a solution only when the errors are small.

IV. CONVENTIONAL QCD SUM RULES FOR SPIN-1/2 DECUPLET STATES

The two sum rules that couple purely to spin-1/2 states are from the Dirac structures $\gamma_\mu p_\nu + \gamma_\nu p_\mu$ and $\gamma_\mu \gamma_\nu + \frac{1}{3}g_{\mu\nu}$. Since they cannot be cast into the $(\gamma_0 A + B)$ form in the rest frame, the parity projection technique

cannot be applied to this case. We have to rely on the conventional sum rule method in which the parities are mixed. The spin-1/2 decuplets are in fact excited states. They are rarely studied in the QCD sum rule method. Here we have an opportunity to isolate them as the ground-state poles. For this reason, even a conventional analysis is beneficial.

The sum rule from the $\gamma_\mu p_\nu + \gamma_\nu p_\mu$ structure is

$$\begin{aligned}
& c_1 [1] L^{4/27} E_2 M^6 + c_2 [b] L^{4/27} E_0 M^2 \\
& + c_3 [m_s a] L^{4/27} E_0 M^2 + c_4 [m_s m_0^2 a] L^{-10/27} \\
& + c_5 [\kappa_v a^2] L^{28/27} + c_6 [m_0^2 a^2] L^{14/27} \frac{1}{M^2} \\
& = \tilde{\alpha}_{1/2-}^2 e^{-m_{1/2-}^2/M^2} + \tilde{\alpha}_{1/2+}^2 e^{-m_{1/2+}^2/M^2}, \quad (21)
\end{aligned}$$

where the coefficients are given by

	c_1	c_2	c_3	c_4	c_5	c_6
Δ	$\frac{1}{240}$	$\frac{5}{1728}$	0	0	$-\frac{1}{18}$	$\frac{7}{216}$
Σ^*	$\frac{1}{240}$	$\frac{5}{1728}$	$\frac{(f_s-4)}{72}$	$\frac{(7-4f_s)}{216}$	$-\frac{(1+2f_s)}{54}$	$\frac{7(1+2f_s)}{648}$
Ξ^*	$\frac{1}{240}$	$\frac{5}{1728}$	$-\frac{(2+f_s)}{36}$	$\frac{(7-f_s)}{36}$	$-\frac{f_s(2+f_s)}{54}$	$\frac{7f_s(1+2f_s)}{648}$
Ω^-	$\frac{1}{240}$	$\frac{5}{1728}$	$-\frac{f_s}{8}$	$\frac{f_s}{24}$	$-\frac{f_s}{18}$	$\frac{7f_s}{216}$

(22)

This sum rule is chiral-even. Since the parity of the lowest state is not known, we retain one term for each parity in the RHS, and let the OPE reveal which one is lower. The sum rule for Δ agrees with that given in Ref. [7], except that the coefficient c_6 has the opposite sign. The sum rules for Σ^* , Ξ^* , and Ω^- are new, to the best of our knowledge. Note that the sum rule for Ω^- reduces to that for Δ if the strange quark is turned off ($m_s = 0$ and $f_s = 1$), as expected. Since they are derived separately, this provides a non-trivial check of the calculation. Another check is provided by the fact that c_5 and c_6 for Σ^* , Ξ^* , and Ω^- coincide with each other in the limit of $f_s = 1$, as expected. The excited state contributions of RHS are modeled using terms on the OPE side surviving $M^2 \rightarrow \infty$ under the assumption of duality, and are represented by the factors $E_n(x) = 1 - e^{-x} \sum_n x^n/n!$ with $x = w^2/M^2$ and w an effective continuum threshold.

The sum rule from the $\gamma_\mu \gamma_\nu + \frac{1}{3}g_{\mu\nu}$ structure, which is chiral-odd, is given by

$$\begin{aligned}
& c_1 [a] L^{16/27} E_1 M^4 + c_2 [m_0^2 a] L^{2/27} E_0 M^2 \\
& + c_3 [m_s b] L^{-8/27} E_0 M^2 + c_4 [a b] L^{16/27} \\
& + c_5 [m_s \kappa_v a^2] L^{16/27} \\
& = -\tilde{\alpha}_{1/2+}^2 m_{1/2+} e^{-m_{1/2+}^2/M^2} \\
& + \tilde{\alpha}_{1/2-}^2 m_{1/2-} e^{-m_{1/2-}^2/M^2}, \quad (23)
\end{aligned}$$

where the coefficients are given by

	c_1	c_2	c_3	c_4	c_5
Δ	$-\frac{1}{36}$	$\frac{1}{18}$	0	$-5/864$	0
Σ^*	$-\frac{(2+f_s)}{108}$	$\frac{(2+f_s)}{54}$	$\frac{1}{288}$	$-\frac{5(2+f_s)}{2592}$	$\frac{(f_s-1)}{18}$
Ξ^*	$-\frac{(1+2f_s)}{108}$	$\frac{(1+2f_s)}{54}$	$\frac{1}{144}$	$-\frac{5(1+2f_s)}{2592}$	$\frac{f_s(f_s-1)}{18}$
Ω^-	$-\frac{f_s}{36}$	$\frac{f_s}{18}$	$\frac{1}{96}$	$-5f_s/864$	0

(24)

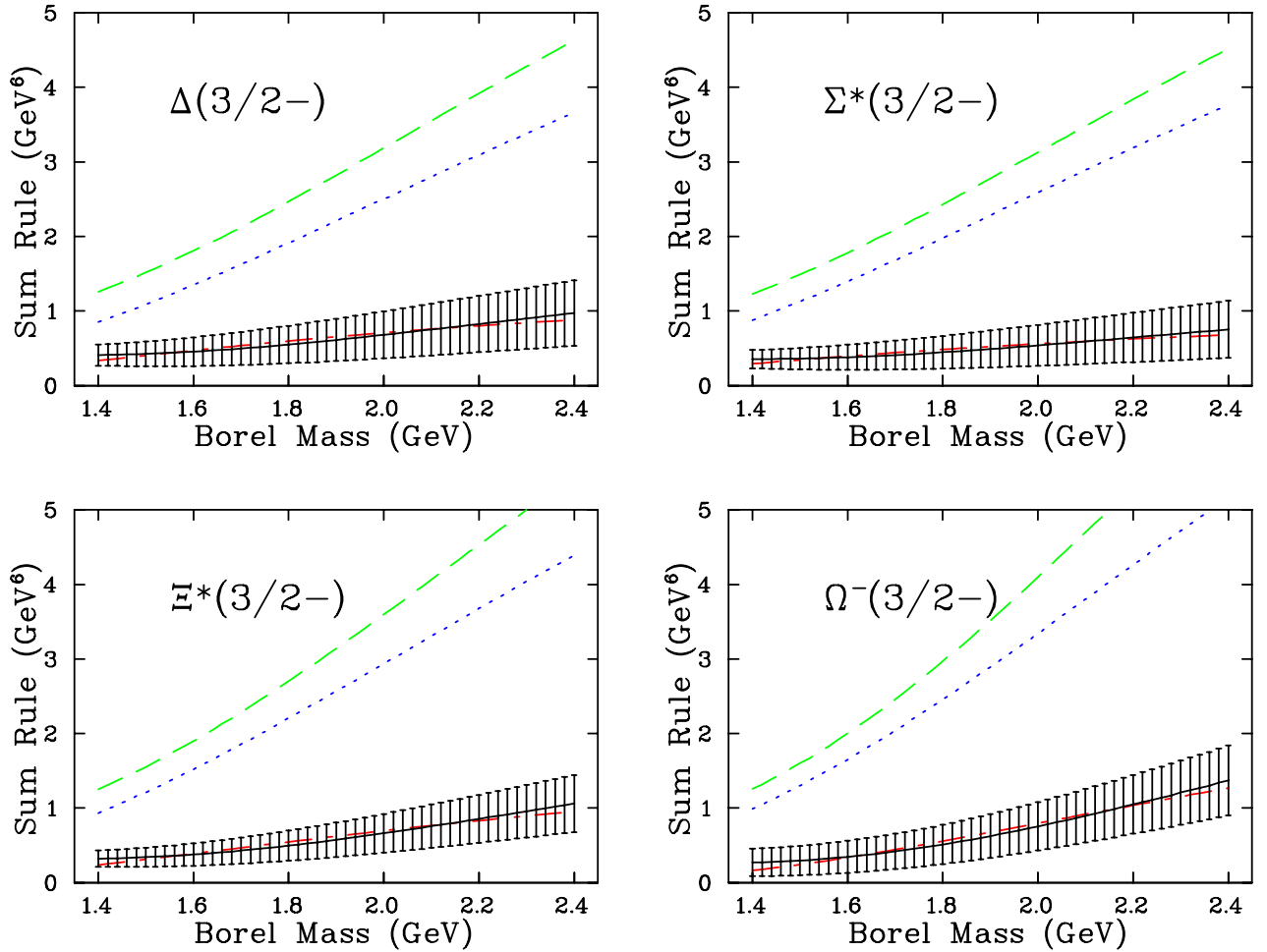


FIG. 2: Matching of the sum rules in Eq. (14) for negative-parity spin-3/2 states. The solid line along with the error band generated via Monte-Carlo is the LHS ($\mathcal{A} - \mathcal{B}$). The dash-dot line, which is barely distinguishable from the solid line, is the RHS (ground-state pole). Also plotted are the individual contributions of term \mathcal{A} (dotted line) and term \mathcal{B} (dashed line).

For Δ , the coefficient c_4 is different in both value and sign from that in Ref. [7]. The sum rules for Σ^* , Ξ^* , and Ω^- are new, as far as we know. Similarly, the sum rule for Ω^- reduces to that for Δ if the strange quark is turned off. Another check is provided by the fact that c_1 , c_2 , c_4 and c_5 for Σ^* , Ξ^* , and Ω^- coincide with each other in the limit of $f_s = 1$.

Now we turn to the analysis of the two sum rules. First, we note that states of opposite parities on the phenomenological side (RHS) are adding up in the chiral-even sum rule, whereas canceling in the chiral-odd sum rule. This is a standard feature of baryon sum rules that leads to the general conclusion that chiral-odd sum rules perform better than chiral-even sum rules in baryon channels. Indeed, we found that the chiral-even sum rule in Eq. (21) is very poor. The leading term is a perturbative contribution, and the sum rule is almost completely saturated by the continuum. No results could be extracted from this sum rule.

On the other hand, the chiral-odd sum rule in Eq. (23) has good convergence. The leading term contains the

non-perturbative quark condensate. The sign of this term (in c_1) indicates that the sum rule is saturated by the positive-parity state: they have the same negative signs, see Eq. (23) and Eq. (24). This is an example of how the parity of the ground-state pole is determined in a mixed-parity sum rule. The dominance of the quark-condensate term is further confirmed in our numerical Monte-Carlo analysis. We are able to perform three-parameter searches. It means that the extracted mass, coupling, continuum threshold can be regarded as the true predictions of the sum rule. The results are given in Table III and the matching of the two sides is shown in Fig. 3. In the observed spectrum, the lowest $\Delta(1/2+)$ is a one-star state at 1.75 GeV, followed by a 4-star state at 1.91 GeV. Our prediction favors the latter as the lowest state in this channel. It casts doubts on the existence of the state at 1.75 GeV. For $\Sigma^*(1/2+)$, the PDG lists a 3-star state at 1.66 GeV, followed by a one-star state at 1.77 GeV, then by a 2-star state at 1.88 GeV. Our result seems to favor the state at 1.88 GeV than the state at 1.66 GeV. The spin-parity situation in the $\Xi^*(1/2+)$

TABLE III: Results for the $1/2+$ branch of the decuplet states from the chiral-odd sum rule Eq. (23) in a three-parameter search. The errors are derived from 500 Monte-Carlo sets based on the uncertainties assigned to the QCD input parameters. Two sets of solutions are provided for each case: the first one using the default errors in the OPE input parameters, while the second using uniform 10% errors. The experimental values are taken from the PDG [15].

	Region	w	$\bar{\alpha}_{1/2+}^2$	Mass	Exp.
	(GeV)	(GeV)	(GeV ⁶)	(GeV)	(GeV)
$\Delta(1/2+)$	1.1 to 2.0	2.41 ± 1.08 2.52 ± 0.80	0.18 ± 0.15 0.18 ± 0.13	2.10 ± 0.54 2.17 ± 0.33	1.91
$\Sigma^*(1/2+)$	1.1 to 2.0	2.15 ± 1.06 2.29 ± 0.83	0.13 ± 0.13 0.13 ± 0.11	1.97 ± 0.62 2.09 ± 0.37	1.88
$\Xi^*(1/2+)$	1.1 to 2.0	2.42 ± 1.11 2.50 ± 1.50	0.16 ± 0.13 0.16 ± 0.12	2.11 ± 0.54 2.19 ± 0.35	1.95(?)
$\Omega^-(1/2+)$	1.1 to 2.0	3.02 ± 1.18 3.11 ± 1.13	0.23 ± 0.14 0.23 ± 0.13	2.37 ± 0.41 2.39 ± 0.32	2.38(?)

channel is not clear in the PDG. It lists two 3-star states with unknown spin-parity, at 1690 MeV and 1950 MeV. Our results is in favor of the 1950 MeV state. In the Ω^- channel, there are 3 states sitting close to each other in the observed spectrum, at 2250, 2380, and 2470 MeV, whose spin and parity are not clear. Our prediction of 2.37 GeV is in the middle of this range, but the accuracy is not enough to clearly identify with one of them.

V. CONCLUSION

We have presented a study of the decuplet family using the method of QCD sum rules. New sum rules are derived for the $3/2+$ and $3/2-$ branches using a parity-projection technique. They are more stable and give a better determination of the mass spectrum than the conventional sum rules. The results for the $3/2-$ branch are new as a consequence of the parity separation. The spin-1/2 sector is investigated using the conventional sum rules method. New sum rules are derived for members that contain the strange quark (Σ^* , Ξ^* , and Ω^-), in addition to a careful re-examination of Δ channel. The chiral-even sum rules of Eq. (21) are dominated by the continuum. No useful information could be extracted from them. The chiral-odd sum rules of Eq. (23) have good convergence and allow the only three-parameter searches in this study. The predicted results provide useful information on the $1/2+$ states from a QCD-based standpoint. We could not find sum rules that are saturated by the $1/2-$ states.

Acknowledgments

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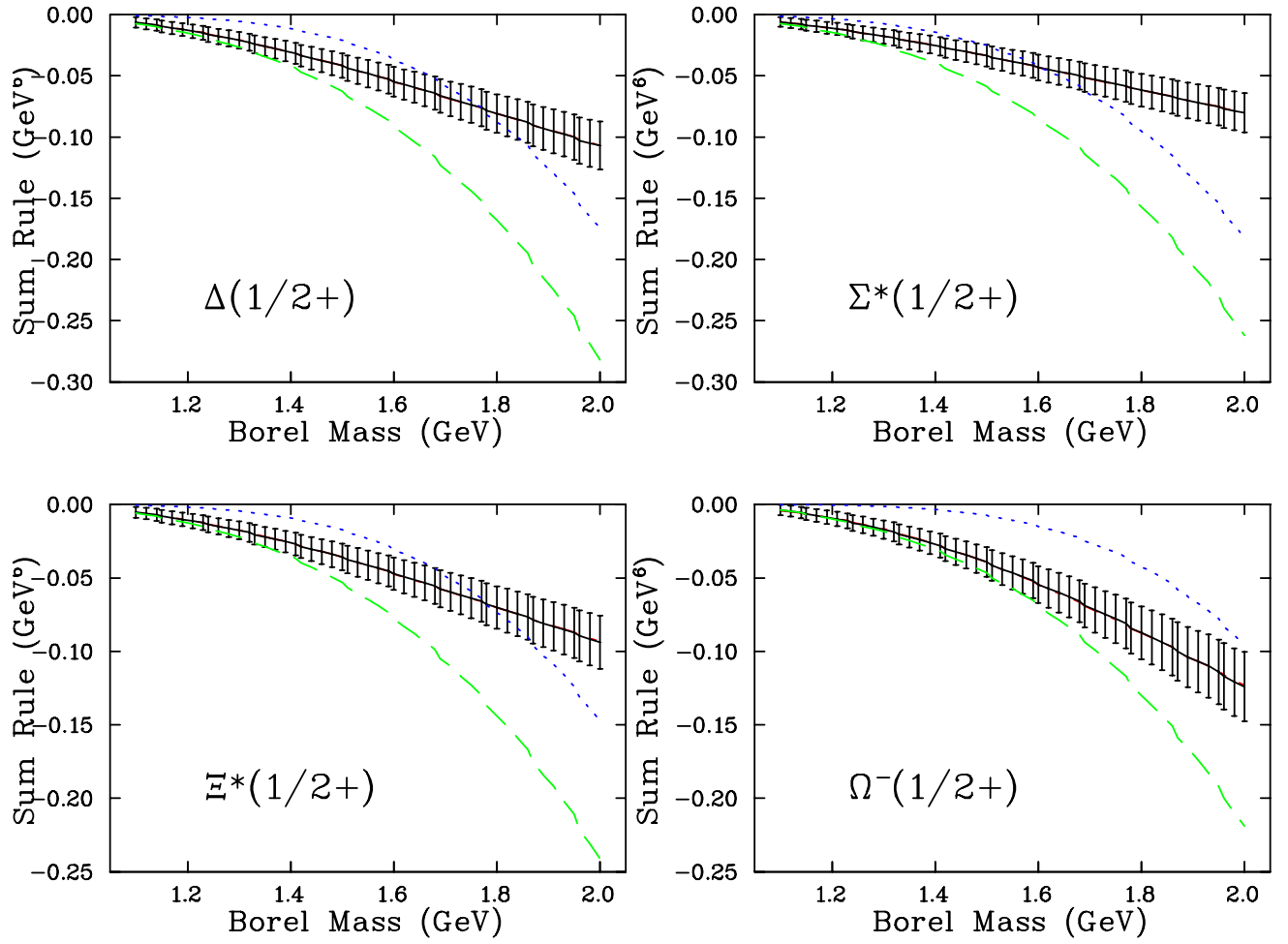


FIG. 3: Matching of the sum rules in Eq. (23) for the spin-1/2 decuplet states. The solid or black line along with the error band generated via Monte-Carlo is the LHS (OPE minus continuum). The dash-dot or red line, which is barely distinguishable from the solid line, is the RHS (ground-state pole). Also plotted are the individual contributions of the OPE (dashed or green line) and the continuum (dotted or blue line).